Letters

An Optimization-Based Design Procedure for Asymmetric Bidirectional Associative Memories

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***Abstract*—In this letter, we consider the problem of designing asymmetric bidirectional associative memories (ABAMs). Based on a newly derived theorem for the ABAM model, we propose an optimization-based design procedure for obtaining an ABAM that can store given bipolar vector pairs with certain error correction properties. Our design procedure consists of generalized eigenvalue problems, which can be efficiently solved by recently developed interior point methods. The validity of the proposed method is illustrated by a design example.**

***Index Terms*—Bidirectional associative memory (BAM), generalized eigenvalue problem.**

I. Introduction

The bidirectional associative memory (BAM) model, which was first introduced by Kosko [1], [2] as an extension of the unidirectional autoassociator of Hopfield [3], is a special class of recurrent neural networks that can store and recall bipolar vector pairs. It is composed of neurons arranged in two layers, the X-layer and the Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer. Through iterations of forward and backward information flows between the two layers, it performs a two-way associative search for stored bipolar vector pairs. Since connection weights of the original Kosko BAMs are encoded via the Hebbian learning rule, they are to inherit the logical symmetry in the sense that the connection weight matrix from the X-layer to the Y-layer is the same as the transpose of the connection weight matrix from the Y-layer to the X-layer. It is now well known that the Kosko BAM suffers from problems such as low storage capacity and low error correction rate, and many efforts have been made for relaxing these problems. An approach along the line of these efforts is to use the asymmetric BAM (ABAM) model [4], in which the logical symmetry of connection weights is abandoned to make better performance possible. The ABAMs designed via the encoding scheme of [4] generally yield better performance than the original Kosko BAMs do. However, the encoding scheme of [4] still has a limitation; it requires linear independence of stored patterns, which limits its storage capacity. In this letter, we propose a new design procedure which is based on the mathematical programming technique called the generalized eigenvalue problem (GEVP). Our approach is quite useful in practice, because GEVPs can be efficiently solved by recently developed interior point methods. For a specific algorithm belonging to these interior point methods, one can refer to the Nesterov and Nemirovskii algorithm [6], for example. A statement and proof of the polynomial-time complexity of the algorithm is given in [6]. Also, an implementation of the algorithm for GEVPs is provided in the LMI Control Toolbox [7] for use with MATLAB. As an optimum searcher for the GEVPs appearing in this letter, we use the function "gevp" of the LMI Control Toolbox. Our focus in this letter is on proposing an improvement over the ABAM of [4]. A related paper [8], which appeared after this work was completed, also reveals and corrects limitations of the ABAM model. It is shown there that a feedforward BAM network designed with a procedure based on a one-shot algorithm can perform perfect bidirectional recall for arbitrarily correlated patterns. Our work and [8] both depend on the effort for maximizing attraction region of each prototype in the training stage. A notable difference of our approach from the method of [8] is that we stick to the conventional architecture while in [8], a feedforward structure is adopted for realizing a new concept of the BAM memories.

Throughout this letter, a bipolar vector is called a *pattern* and the usual Hamming distance between two patterns x and x^* is denoted by $H(x, x^*)$.

II. Main Theorem and GEVP-Based Design of ABAMs

Consider the ABAM model with n neurons in the X-layer and p neurons in the Y-layer. Its iterative evolution, which follows $x^{\text{new}} = \text{sgn}(Wy^{\text{old}})$ and $y^{\text{new}} = \text{sgn}(Vx^{\text{old}})$ alternately, yields a meaningful result when the state of the network, (x, y), reaches a stable pattern pair (x^*, y^*) satisfying $x^* = \text{sgn}(Wy^*)$ and $y^* = \text{sgn}(Vx^*)$. Note that if both $x_i^*(\sum_{j=1}^p w_{ij}y_j^*) > 0, \forall\, i$, and $y_j^*(\sum_{i=1}^n v_{ji}x_i^*) > 0$, $\forall\, j$, hold, then (x^*, y^*) must be a stable pattern pair. In the following, we present a theorem, which will play a central role in our design of the ABAM memories.

Theorem 1: Given a desired stable pattern pair $(x^*, y^*) \in \{-1, 1\}^{n+p}$, let the connection weight matrix from the Y-layer to the X-layer, W, of the ABAM satisfy

$$x_i^*\left(\sum_{j=1}^p w_{ij}y_j^*\right) > 2d_i \max_j |w_{ij}| \tag{1}$$

for some $d_i > 0$. If the Y-layer has $y \in \{-1, 1\}^p$ satisfying $H(y, y^*) \le d_i$ as its vector value at time step t, then at the next time step, the ith neuron of the X-layer will have the correct value, i.e., $x_i(t+1) = x_i^*$.

Proof: Let $y \in \{-1, 1\}^p$ be any pattern satisfying $H(y, y^*) \le d_i$, and let the vector value of the Y-layer at time t be y. Since $\delta \triangleq y - y^*$ satisfies

$$\left|\sum_{j=1}^p w_{ij}\delta_j\right| \le 2d_i \max_j |w_{ij}|$$

we have

$$\begin{aligned} x_i^*\left(\sum_{j=1}^p w_{ij}y_j\right) &= x_i^*\left(\sum_{j=1}^p w_{ij}y_j^* + \sum_{j=1}^p w_{ij}\delta_j\right) \\ &\ge x_i^*\left(\sum_{j=1}^p w_{ij}y_j^*\right) - \left|\sum_{j=1}^p w_{ij}\delta_j\right| \\ &\ge x_i^*\left(\sum_{j=1}^p w_{ij}y_j^*\right) - 2d_i \max_j |w_{ij}| \\ &> 0. \end{aligned}$$

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TABLE I
AVERAGE ERROR CORRECTION PROBABILITIES, Prob(r), $r = 0, \ldots, 4$, OF DESIGNED ABAMs

	$r=0$	$r=1$	$r=2$	$r=3$	$r=4$
Proposed method	1.00	0.889	0.566	0.422	0.306
Method of Xu et al. [4]	1.00	0.804	0.527	0.294	0.186

This implies that x_i^* and $\sum_{j=1}^{p} w_{ij} y_j$ have the same sign. Thus, we have $x_i(t+1) = \text{sgn}(\sum_{j=1}^{p} w_{ij} y_j) = x_i^*$. □

Similarly, we can show that if the connection weight matrix from the X-layer to the Y-layer, V, of the ABAM satisfies

$$y_j^* \left(\sum_{i=1}^{n} v_{ji} x_i^* \right) > 2 g_j \max_i |v_{ji}| \tag{2}$$

for some $g_j > 0$, then the same kind of error correction capability is guaranteed at the jth neuron of the Y-layer. From the above, we note that with each of the d_i and g_j getting larger, the neighboring region of the desired pattern pair where the one-step error correction will be guaranteed becomes larger. Furthermore, if W and V of the ABAM satisfy (1) and (2) with $d_i > 0$, $i = 1, \ldots, n$, and $g_j > 0$, $j = 1, \ldots, p$, it is ensured that (x^*, y^*) is stored as a stable pattern pair of the ABAM. Therefore, by maximizing each d_i and each g_j, one can design an ABAM not only capable of storing given pattern pairs, but also efficient in error correction. This design strategy can be summed up as the following: Given desired pattern pairs $\{(x^{(k)}, y^{(k)}) | k = 1, \ldots, m\}$, find connection weight matrices, W and V, of the ABAM by solving

$$\begin{aligned} &\max\ d_i (> 0) \\ &\text{s.t. } x_i^{(k)} \left(\sum_{j=1}^{p} w_{ij} y_j^{(k)} \right) > 2 d_i \max_j |w_{ij}|, \qquad k = 1, \ldots, m \end{aligned} \tag{3}$$

for each $i \in \{1, \ldots, n\}$, and solving

$$\begin{aligned} &\max\ g_j (> 0) \\ &\text{s.t. } y_j^{(k)} \left(\sum_{i=1}^{n} v_{ji} x_i^{(k)} \right) > 2 g_j \max_i |v_{ji}|, \qquad k = 1, \ldots, m \end{aligned} \tag{4}$$

for each $j \in \{1, \ldots, p\}$. Utilizing additional positive variables p_is and q_js, we can convert problems (3) and (4) into the following GEVP, respectively. (For details on GEVPs, see, e.g., [5]):

$$\begin{aligned} &\min\ (-d_i)(< 0) \\ &\text{s.t. } -x_i^{(k)} \left(\sum_{j=1}^{p} w_{ij} y_j^{(k)} \right) < (-d_i)(2p_i), \quad k = 1, \ldots, m, \\ &\qquad -p_i < w_{ij} < p_i, \quad j = 1, \ldots, p. \end{aligned} \tag{5}$$

$$\begin{aligned} &\min\ (-g_j)(< 0) \\ &\text{s.t. } -y_j^{(k)} \left(\sum_{i=1}^{n} v_{ji} x_i^{(k)} \right) < (-g_j)(2q_j), \quad k = 1, \ldots, m, \\ &\qquad -q_j < v_{ji} < -q_j, \quad i = 1, \ldots, n. \end{aligned} \tag{6}$$

The magnitudes of connection weights can be maintained within a reasonable bound by imposing further constraints: $L < p_i < U, \forall i$, and $L < q_j < U, \forall j$. Note that resulting optimization problems are still GEVPs. Finally, note that the d_i and g_j obtained by solving (5) and (6) yield the following information about retrieval errors: If $\min_{1 \le i \le n} d_i$ and $\min_{1 \le j \le p} g_j$ are both greater than $r \in \{0, 1, \ldots\}$, then any trajectory starting from a bipolar initial condition vector r-bits away from a stored pattern will never result in the retrieval error.

III. A DESIGN EXAMPLE

To demonstrate the applicability of the proposed method, we consider an example, in which we wish to design an ABAM that can associate each uppercase English letter in the color graphics adapter (CGA) font with its lowercase counterpart. Solving the corresponding GEVPs with $L = 1$ and $U = 2$, we obtained an ABAM memory. Given a noisy uppercase (or lowercase) letter, this memory is expected to recall the corresponding letter pair. To compare with an existing method, we also considered the ABAM memory designed by the method of Xu *et al.* [4] for the same pattern pairs. As an index for performance comparison, we used the average error correction probability Prob(r), which is defined as follows: Given the pair of an uppercase letter $x^{(k)}$ and an integer $r \in \{0, 1, \ldots\}$, we define the error correction probability $P(x^{(k)}, r)$ as the probability that the trajectory starting from a bipolar initial condition vector on the surface of the Hamming ball of radius r around $x^{(k)}$ can successfully reach the correct pattern pair $(x^{(k)}, y^{(k)})$. The average error correction probability Prob(r) is the average of the $P(x^{(k)}, r)$ over all the $x^{(k)}$. Shown in Table I are estimates for the Prob(r) obtained via simulations, in which the error was considered to be uncorrectable if the original desired pattern pair could not be recovered in 100 time steps. The contents of the table show that the memory designed by the proposed method is superior to the other one.

IV. CONCLUSION

In this letter, we proposed a GEVP-based design procedure for asymmetric bidirectional associative memories. Since efficient interior point methods that can solve the GEVPs arising in the procedure within a given tolerance are readily available, the procedure is very useful in practice. A design example was presented for an illustration, and simulation results showed that the proposed method can be a promising choice for the design of ABAMs.

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